

The MSSM without Gluinos; an Effective Field Theory for the Stop Sector

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Abstract

In this article we study the MSSM with stops and Higgs scalars much lighter than gluinos and squarks of the first two generations. In this setup, one should use an effective field theory with partial supersymmetry in which the gluino and heavy squarks are integrated out in order to connect SUSY parameters (given at a high scale) to observables in the stop sector. In the construction of this effective theory, valid below the gluino mass scale, we take into account $O(\alpha_3)$ and $O(Y_{t,b}^2)$ effects and calculate the matching as well as the renormalization group evolution. As a result, the running of the parameters for the stop sector is modified with respect to the full MSSM and SUSY relations between parameters are broken. We show that for some couplings sizable numerical differences exist between the effective field theory approach and the naive calculation based on the MSSM running.

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I. INTRODUCTION

There are several theoretical arguments for a light stop in supersymmetric theories. Foremost, in natural supersymmetry (SUSY) light stops are required to cancel the quadratic divergence of the Higgs mass originating from the self-energy involving a top quark, while the other supersymmetric partners can be much heavier [1, 2] due to the smaller couplings to the Higgs. Moreover, the renormalization group equations (RGE) of the minimal supersymmetric standard model (MSSM) generically drive the bilinear mass term parameters of the third generation squarks to lower values (compared to the first two generations) due to their non-negligible Yukawa couplings [3–8].

Although the measured Higgs mass of around 125 GeV [9, 10] prefers rather heavy (around the TeV scale) [11–13] rather than light stops in the MSSM, this is not necessarily the case in the NMSSM [14], in λ SUSY models [15] or in supersymmetric models with additional D-term [16] or F-term [17] contributions to the scalar potential. Also large (or even maximal [18–20]) stop mixing angles help to get the right Higgs mass with rather light stops.

LHC searches for top squarks (using simplified models) set a lower bound on its mass of around $m_{\tilde{t}_1} = 300$ GeV, which however heavily depends on the neutralino mass. Depending on the stop and the neutralino mass, different decay modes are studied. For the decay channel $\tilde{t}_1 \rightarrow t \tilde{\chi}_1^0$ [21–23], the limits are quite stringent, even though for light neutralinos very light stops can not be excluded due to the high $t\bar{t}$ -background [24]. The three-body decay $\tilde{t}_1 \rightarrow Wb\tilde{\chi}_1^0$ was analyzed theoretically in [25] and experimentally in [26]. Finally the decay $\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$ and the less important four-body decay $\tilde{t}_1 \rightarrow \tilde{\chi}_1^0 d_i f \bar{f}'$ are treated in [27–29] and constraints were derived by the ATLAS collaboration from the monojet analysis in [30]. Some bounds can be avoided in kinematic boundary regions or once non-minimal flavour violation is included. However, recently efforts of closing these gaps have been made [31–34] and stops should in general not be lighter than 300 GeV. Nevertheless, the mass bound for the stop is still weaker than the strong bounds on the squark masses of the first two generations and also on the gluino mass [35, 36].

If the gluino (or the squarks of the first two generations [2, 37]) is much heavier than the stops, an effective theory (EFT) with partial SUSY must be constructed in which the gluino (squarks) is integrated out [38] ([39, 40]). The construction of this effective theory for the stop sector is the goal of this article. Assuming a common large mass of order M for the gluino and the squarks of the first two generations, we compute the matching condition between the full MSSM and the effective theory, including one-loop contributions which are enhanced by powers of M . Furthermore, since some supermultiplets are partially integrated out in the effective theory, the supersymmetric relations between gauge/Yukawa couplings,

gaugino/Higgsino couplings and four-scalar couplings are broken in the effective theory by radiative corrections. Therefore, these couplings in the effective theory have an independent renormalization group evolution, as discussed in [38, 41–48] mainly for the gaugino-matter couplings.

This article is structured as follows: In the next section we establish our effective theory for the stop sector and calculate the matching as well as the running of the relevant parameters at order $\alpha_3 = g_3^2/(4\pi)$, Y_t^2 and Y_b^2 (neglecting $O(g_1^2)$, $O(g_2^2)$ and Higgs self-coupling effects). This section is followed by a numerical analysis in Sec. III. Finally we conclude in Sec. IV.

II. THE EFFECTIVE THEORY FOR THE STOP SECTOR

The aim of this section is to construct the effective theory for the MSSM stop sector, including $O(\alpha_3, Y_{t,b}^2)$ and enhanced effects. As noted before, we assume that the gluino and the squarks of the first two generations are much heavier, with masses of the order M , than the stops, the Higgs scalars and the Higgsinos. The left-handed sbottom is also assumed to be light such that it remains in the effective theory, forming an $SU(2)$ multiplet with the left-handed stop. However, we assume that the right-handed sbottom is heavy, with the mass of the order M . Therefore, we consider the following effective Lagrangian which is valid below the scale M ,

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & \mathcal{L}_K - \bar{m}_2^2 H_u^\dagger H_u - \bar{m}_1^2 H_d^\dagger H_d - V(H_u, H_d) \\
& + \bar{m}_{12}^2 H_d \cdot H_u - \bar{\mu} \tilde{H}_U \cdot \tilde{H}_D + (h.c.) \\
& - \bar{m}_Q^2 \tilde{q}_L^\dagger \tilde{q}_L - \bar{m}_t^2 \tilde{t}_R^\dagger \tilde{t}_R \\
& - \bar{Y}_t \bar{t}_R q_{3L} \cdot H_u - \bar{Y}_b \bar{b}_R H_d \cdot q_{3L} + (h.c.) \\
& - \lambda_1^u (\tilde{q}_L^\dagger \tilde{q}_L) (H_u^\dagger H_u) - \lambda_2^u (\tilde{q}_L^\dagger H_u) (H_u^\dagger \tilde{q}_L) - \lambda_3^u (\tilde{t}_R^\dagger \tilde{t}_R) (H_u^\dagger H_u) \\
& - \lambda_1^d (\tilde{q}_L^\dagger \tilde{q}_L) (H_d^\dagger H_d) - \lambda_2^d (\tilde{q}_L^\dagger H_d) (H_d^\dagger \tilde{q}_L) - \lambda_3^d (\tilde{t}_R^\dagger \tilde{t}_R) (H_d^\dagger H_d) \\
& - \lambda_4 (\tilde{q}_{Li}^\dagger \tilde{q}_{Li}) (\tilde{q}_{Lj}^\dagger \tilde{q}_{Lj}) - \lambda_5 (\tilde{q}_{Li}^\dagger \tilde{q}_{Lj}) (\tilde{q}_{Lj}^\dagger \tilde{q}_{Li}) \\
& - \lambda_6 (\tilde{q}_{Li}^\dagger \tilde{q}_{Li}) (\tilde{t}_R^\dagger \tilde{t}_R) - \lambda_7 (\tilde{q}_{Li}^\dagger \tilde{t}_R) (\tilde{t}_R^\dagger \tilde{q}_{Li}) - \lambda_8 (\tilde{t}_R^\dagger \tilde{t}_R) (\tilde{t}_R^\dagger \tilde{t}_R) \\
& - \bar{A}_t \tilde{t}_R^\dagger \tilde{q}_L \cdot H_u + \bar{\mu}_t \tilde{t}_R^\dagger H_d^\dagger \tilde{q}_L + (h.c.) \\
& - \bar{Y}_{q_{3L}} \tilde{t}_R^\dagger q_{3L} \cdot \tilde{H}_U - \bar{Y}_{t_R} \bar{t}_R \tilde{q}_L \cdot \tilde{H}_U - \bar{Y}_{b_R} \bar{b}_R \tilde{H}_D \cdot \tilde{q}_L + (h.c.). \tag{1}
\end{aligned}$$

with partial supersymmetry. Here \mathcal{L}_K denotes the kinetic terms and gauge interactions, and $V(H_u, H_d)$ denotes the quartic couplings of the Higgs doublets (H_u, H_d) . For the interactions involving four squarks, the $SU(3)$ color indices are contracted within the parentheses.

Similarly, the $SU(2)$ indices in the two-squark-two-Higgs interactions are contracted within the parentheses. i, j are the $SU(2)$ indices and the dot denotes the contraction of $SU(2)$ indices as $A \cdot B = A_1 B_2 - A_2 B_1$. For simplicity, we also assume that the electroweak gauginos and sleptons are heavy. However, since we neglect $O(g_1^2)$, $O(g_2^2)$ effects in the following, relaxing this assumption would leave our RGEs unchanged. We also ignore the loop-induced non-holomorphic Higgs-quark couplings $\bar{t}_R H_d^\dagger q_{3L}$ and $\bar{b}_R H_u^\dagger q_{3L}$ since they are suppressed by the ratio μ/M , which is very small by our assumption.

A. Tree-level matching

At the matching scale M the Lagrangian of eq. (1) has to be compared to the one of the full MSSM (see for example [49–52]) which originates from the superpotential

$$W = Y_t T^c Q \cdot H_u + Y_b B^c H_d \cdot Q + \mu H_u \cdot H_d, \quad (2)$$

the soft SUSY breaking terms

$$\begin{aligned} V_{\text{soft}} = & m_Q^2 \tilde{q}_L^\dagger \tilde{q}_L + m_{\tilde{t}}^2 \tilde{t}_R^\dagger \tilde{t}_R + m_{H_d}^2 H_d^\dagger H_d + m_{H_u}^2 H_u^\dagger H_u + m_{\tilde{b}_R}^2 \tilde{b}_R^\dagger \tilde{b}_R \\ & + A_t \tilde{t}_R^\dagger \tilde{q}_L \cdot H_u + A_b \tilde{b}_R^\dagger H_d \cdot \tilde{q}_L - m_{H_d H_u}^2 H_d \cdot H_u + (h.c.), \end{aligned} \quad (3)$$

and the D terms

$$V_D = \frac{g_3^2}{2} \left(\tilde{q}_L^\dagger T^A \tilde{q}_L - \tilde{t}_R^\dagger T^A \tilde{t}_R - \tilde{b}_R^\dagger T^A \tilde{b}_R \right)^2, \quad (4)$$

where T^A are the generators of $SU(3)$ in the fundamental representation.

The matching conditions for the bilinear terms and the trilinear couplings are

$$\bar{Y}_t = Y_t, \quad \bar{Y}_b = Y_b, \quad \bar{Y}_{q_{3L}} = Y_t, \quad \bar{Y}_{t_R} = Y_t, \quad \bar{Y}_{b_R} = Y_b, \quad \bar{A}_t = A_t, \quad (5)$$

$$\bar{\mu} = \mu, \quad \bar{\mu}_t = \mu Y_t, \quad \bar{m}_2^2 = m_{H_u}^2 + \mu^2, \quad \bar{m}_1^2 = m_{H_d}^2 + \mu^2, \quad (6)$$

$$\bar{m}_{12}^2 = m_{H_d H_u}^2, \quad \bar{m}_{\tilde{Q}}^2 = m_{\tilde{Q}}^2, \quad \bar{m}_{\tilde{t}}^2 = m_{\tilde{t}}^2. \quad (7)$$

The couplings between squarks and Higgs bosons are generated by F- and D-terms in the MSSM Lagrangian. At the scale M , they are given by

$$\lambda_1^u = Y_t^2, \quad \lambda_2^u = -Y_t^2, \quad \lambda_3^u = Y_t^2, \quad (8)$$

$$\lambda_1^d = Y_b^2, \quad \lambda_2^d = -Y_b^2, \quad \lambda_3^d = 0, \quad (9)$$

$$\lambda_4 = -\frac{1}{12}g_3^2, \quad \lambda_5 = \frac{1}{4}g_3^2, \quad \lambda_6 = \frac{1}{6}g_3^2, \quad (10)$$

$$\lambda_7 = -\frac{1}{2}g_3^2 + Y_t^2, \quad \lambda_8 = \frac{1}{6}g_3^2, \quad (11)$$

keeping only Yukawa couplings and g_3 .

B. 1-loop matching

For the matching, we need to include the one-loop effects enhanced by powers of M since their contributions may be comparable to the tree level ones shown in the previous subsection. They can only appear in bilinear and trilinear terms, as seen by dimensional analysis. The bilinear terms receive the following shifts at the matching scale $\mu = M$

$$\Delta\bar{m}_2^2 = 0, \quad \Delta\bar{m}_1^2 = -\frac{3}{16\pi^2}(Y_b^2 m_{\tilde{b}_R}^2 + A_b^2) \left(1 - \log\left(\frac{m_{\tilde{b}_R}^2}{M^2}\right)\right), \quad (12)$$

$$\Delta\bar{m}_Q^2 = -\frac{1}{16\pi^2}(Y_b^2 m_{\tilde{b}_R}^2 + A_b^2)(1 - \log\left(\frac{m_{\tilde{b}_R}^2}{M^2}\right)) + \frac{\alpha_3 C_F}{\pi} m_{\tilde{g}}^2 \left(1 - \log\left(\frac{m_{\tilde{g}}^2}{M^2}\right)\right), \quad (13)$$

$$\Delta\bar{m}_t^2 = \frac{\alpha_3 C_F}{\pi} m_{\tilde{g}}^2 \left(1 - \log\left(\frac{m_{\tilde{g}}^2}{M^2}\right)\right), \quad (14)$$

$$\Delta\bar{m}_{12}^2 = -\frac{3A_b \mu Y_b}{16\pi^2} \left(1 - \log\left(\frac{m_{\tilde{b}_R}^2}{M^2}\right)\right), \quad (15)$$

$$\Delta\bar{\mu} = 0. \quad (16)$$

For the trilinear term the shift reads

$$\Delta\bar{A}_t = -\frac{A_b Y_t Y_b}{16\pi^2} \left(1 - \log\left(\frac{m_{\tilde{b}_R}^2}{M^2}\right)\right) - \frac{\alpha_3 C_F}{\pi} m_{\tilde{g}} Y_t \left(1 - \log\left(\frac{m_{\tilde{g}}^2}{M^2}\right)\right), \quad (17)$$

$$\Delta\bar{\mu}_t = 0. \quad (18)$$

All the other parameters relevant for the stop sector are dimensionless and therefore do not receive any M enhanced corrections.

C. Renormalization group evolution

The running of the full MSSM parameters [3–7] is known at the two-loop level [8, 53–56]. Here we give the one-loop beta functions to $\mathcal{O}(\alpha_3, Y_{t,b}^2)$ for the parameters of our effective theory in eq. (1). The corresponding results for the full MSSM are summarized in the appendix. For the strong coupling constant we have ($t \equiv \log \mu$, μ is the renormalization scale)

$$16\pi^2 \frac{d}{dt} \bar{g}_3 = \left(-7 + \frac{1}{2}\right) \bar{g}_3^3, \quad (19)$$

where the first term on the right hand side is the SM contribution. The effective quark-quark-Higgs Yukawa couplings evolve according to

$$16\pi^2 \frac{d}{dt} \bar{Y}_t = \bar{Y}_t \left[-8\bar{g}_3^2 + \frac{9}{2}\bar{Y}_t^2 + \frac{1}{2}\bar{Y}_b^2 + \bar{Y}_{t_R}^2 + \frac{1}{2}\bar{Y}_{q_{3L}}^2 \right], \quad (20)$$

$$16\pi^2 \frac{d}{dt} \bar{Y}_b = \bar{Y}_b \left[-8\bar{g}_3^2 + \frac{1}{2}\bar{Y}_t^2 + \frac{9}{2}\bar{Y}_b^2 + \bar{Y}_{b_R}^2 + \frac{1}{2}\bar{Y}_{q_{3L}}^2 \right], \quad (21)$$

while the evolution of the ones entering the Higgsino-quark-squark vertex is determined by

$$16\pi^2 \frac{d}{dt} \bar{Y}_{q_{3L}} = \bar{Y}_{q_{3L}} \left[-4\bar{g}_3^2 + \frac{1}{2}\bar{Y}_t^2 + \frac{1}{2}\bar{Y}_b^2 + 4\bar{Y}_{q_{3L}}^2 + \frac{3}{2}\bar{Y}_{t_R}^2 \right], \quad (22)$$

$$16\pi^2 \frac{d}{dt} \bar{Y}_{t_R} = \bar{Y}_{t_R} \left[-4\bar{g}_3^2 + \bar{Y}_t^2 + \frac{3}{2}\bar{Y}_{q_{3L}}^2 + \frac{7}{2}\bar{Y}_{t_R}^2 + \bar{Y}_{b_R}^2 \right], \quad (23)$$

$$16\pi^2 \frac{d}{dt} \bar{Y}_{b_R} = \bar{Y}_{b_R} \left[-4\bar{g}_3^2 + \bar{Y}_b^2 + \bar{Y}_{t_R}^2 + \frac{7}{2}\bar{Y}_{b_R}^2 \right]. \quad (24)$$

For the Higgs mass parameters we find

$$16\pi^2 \frac{d}{dt} \bar{m}_2^2 = 6\bar{Y}_t^2 \bar{m}_2^2 + 6(2\lambda_1^u + \lambda_2^u) \bar{m}_{\tilde{Q}}^2 + 6\lambda_3^u \bar{m}_t^2 + 6\bar{A}_t^2, \quad (25)$$

$$16\pi^2 \frac{d}{dt} \bar{m}_1^2 = 6\bar{Y}_b^2 \bar{m}_1^2 + 6(2\lambda_1^d + \lambda_2^d) \bar{m}_{\tilde{Q}}^2 + 6\lambda_3^d \bar{m}_t^2 + 6\bar{\mu}_t^2, \quad (26)$$

$$16\pi^2 \frac{d}{dt} \bar{m}_{12}^2 = 3(\bar{Y}_t^2 + \bar{Y}_b^2) \bar{m}_{12}^2 + 6\bar{\mu}_t \bar{A}_t, \quad (27)$$

and for the bilinear squark mass terms

$$\begin{aligned} 16\pi^2 \frac{d}{dt} \bar{m}_{\tilde{Q}}^2 = & [-8\bar{g}_3^2 + 2\bar{Y}_{t_R}^2 + 2\bar{Y}_{b_R}^2 + 28\lambda_4 + 20\lambda_5] \bar{m}_{\tilde{Q}}^2 + (6\lambda_6 + 2\lambda_7) \bar{m}_{\tilde{t}}^2 \\ & + (4\lambda_1^u + 2\lambda_2^u) \bar{m}_2^2 + (4\lambda_1^d + 2\lambda_2^d) \bar{m}_1^2 \\ & + 2(\bar{A}_t^2 + \bar{\mu}_t^2) - 4(\bar{Y}_{t_R}^2 + \bar{Y}_{b_R}^2) \bar{\mu}^2, \end{aligned} \quad (28)$$

$$\begin{aligned} 16\pi^2 \frac{d}{dt} \bar{m}_{\tilde{t}}^2 = & [-8\bar{g}_3^2 + 4\bar{Y}_{q_{3L}}^2 + 16\lambda_8] \bar{m}_{\tilde{t}}^2 + (12\lambda_6 + 4\lambda_7) \bar{m}_{\tilde{Q}}^2 \\ & + 4\lambda_3^u \bar{m}_2 + 4\lambda_3^d \bar{m}_1 + 4(\bar{A}_t^2 + \bar{\mu}_t^2) - 8\bar{Y}_{q_{3L}}^2 \bar{\mu}^2. \end{aligned} \quad (29)$$

The Higgsino mass in the effective theory evolves as

$$16\pi^2 \frac{d}{dt} \bar{\mu} = \frac{3}{2}(\bar{Y}_{q_{3L}}^2 + \bar{Y}_{t_R}^2 + \bar{Y}_{b_R}^2) \bar{\mu}, \quad (30)$$

and the effective trilinear $H\tilde{q}\tilde{q}$ coupling as

$$16\pi^2 \frac{d}{dt} \bar{A}_t = \bar{A}_t [-8\bar{g}_3^2 + 2\bar{Y}_{q_{3L}}^2 + \bar{Y}_{t_R}^2 + \bar{Y}_{b_R}^2 + 3\bar{Y}_t^2 + 2\lambda_1^u - 2\lambda_2^u + 2\lambda_3^u + 2\lambda_6 + 6\lambda_7], \quad (31)$$

$$\begin{aligned} 16\pi^2 \frac{d}{dt} \bar{\mu}_t = & \bar{\mu}_t [-8\bar{g}_3^2 + 2\bar{Y}_{q_{3L}}^2 + \bar{Y}_{t_R}^2 + \bar{Y}_{b_R}^2 + 3\bar{Y}_b^2 + 2\lambda_1^d + 4\lambda_2^d + 2\lambda_3^d + 2\lambda_6 + 6\lambda_7] \\ & + 4\bar{Y}_{q_{3L}} \bar{Y}_{b_R} \bar{Y}_b \bar{\mu}. \end{aligned} \quad (32)$$

Finally for the quartic $HH\tilde{q}\tilde{q}$ and $\tilde{q}\tilde{q}\tilde{q}\tilde{q}$ couplings one obtains

$$16\pi^2 \frac{d}{dt} \lambda_1^u = 4(\lambda_1^u)^2 + 2(\lambda_2^u)^2 + 28\lambda_1^u \lambda_4 + 20\lambda_1^u \lambda_5 + 12\lambda_2^u \lambda_4 + 4\lambda_2^u \lambda_5 + 6\lambda_3^u \lambda_6 \\ + 2\lambda_3^u \lambda_7 + (-8\bar{g}_3^2 + 6\bar{Y}_t^2 + 2\bar{Y}_{t_R}^2 + 2\bar{Y}_{b_R}^2) \lambda_1^u - 4\bar{Y}_{t_R}^2 \bar{Y}_t^2, \quad (33)$$

$$16\pi^2 \frac{d}{dt} \lambda_2^u = 8\lambda_1^u \lambda_2^u + 4(\lambda_2^u)^2 + 4\lambda_2^u \lambda_4 + 12\lambda_2^u \lambda_5 \\ + (-8\bar{g}_3^2 + 6\bar{Y}_t^2 + 2\bar{Y}_{t_R}^2 + 2\bar{Y}_{b_R}^2) \lambda_2^u, \quad (34)$$

$$16\pi^2 \frac{d}{dt} \lambda_3^u = 12\lambda_1^u \lambda_6 + 6\lambda_2^u \lambda_6 + 4\lambda_1^u \lambda_7 + 2\lambda_2^u \lambda_7 + 4(\lambda_3^u)^2 + 16\lambda_3^u \lambda_8 \\ + (-8\bar{g}_3^2 + 6\bar{Y}_t^2 + 4\bar{Y}_{q_{3L}}^2) \lambda_3^u - 4\bar{Y}_{q_{3L}}^2 \bar{Y}_t^2, \quad (35)$$

$$16\pi^2 \frac{d}{dt} \lambda_1^d = 4(\lambda_1^d)^2 + 2(\lambda_2^d)^2 + 28\lambda_1^d \lambda_4 + 20\lambda_1^d \lambda_5 + 12\lambda_2^d \lambda_4 + 4\lambda_2^d \lambda_5 \\ + 6\lambda_3^d \lambda_6 + 2\lambda_3^d \lambda_7 + (-8\bar{g}_3^2 + 6\bar{Y}_b^2 + 2\bar{Y}_{t_R}^2 + 2\bar{Y}_{b_R}^2) \lambda_1^d - 4\bar{Y}_{b_R}^2 \bar{Y}_b^2, \quad (36)$$

$$16\pi^2 \frac{d}{dt} \lambda_2^d = 8\lambda_1^d \lambda_2^d + 4(\lambda_2^d)^2 + 4\lambda_2^d \lambda_4 + 12\lambda_2^d \lambda_5 \\ + (-8\bar{g}_3^2 + 6\bar{Y}_b^2 + 2\bar{Y}_{t_R}^2 + 2\bar{Y}_{b_R}^2) \lambda_2^d, \quad (37)$$

$$16\pi^2 \frac{d}{dt} \lambda_3^d = 12\lambda_1^d \lambda_6 + 6\lambda_2^d \lambda_6 + 4\lambda_1^d \lambda_7 + 2\lambda_2^d \lambda_7 + 4(\lambda_3^d)^2 + 16\lambda_3^d \lambda_8 \\ + (-8\bar{g}_3^2 + 6\bar{Y}_b^2 + 4\bar{Y}_{q_{3L}}^2) \lambda_3^d - 4\bar{Y}_{q_{3L}}^2 \bar{Y}_b^2, \quad (38)$$

$$16\pi^2 \frac{d}{dt} \lambda_4 = 2(\lambda_1^u)^2 + 2\lambda_1^u \lambda_2^u + 2(\lambda_1^d)^2 + 2\lambda_1^d \lambda_2^d + 40\lambda_4^2 + 40\lambda_4 \lambda_5 + 12\lambda_5^2 + 3\lambda_6^2 \\ + 2\lambda_6 \lambda_7 + (-16\bar{g}_3^2 + 4(\bar{Y}_{t_R}^2 + \bar{Y}_{b_R}^2)) \lambda_4 + \frac{11}{12} \bar{g}_3^4, \quad (39)$$

$$16\pi^2 \frac{d}{dt} \lambda_5 = (\lambda_2^u)^2 + (\lambda_2^d)^2 + 24\lambda_4 \lambda_5 + 20\lambda_5^2 + \lambda_7^2 + (-16\bar{g}_3^2 + 4(\bar{Y}_{t_R}^2 + \bar{Y}_{b_R}^2)) \lambda_5 \\ - 2(\bar{Y}_{t_R}^4 + \bar{Y}_{b_R}^4) + \frac{5}{4} \bar{g}_3^4, \quad (40)$$

$$16\pi^2 \frac{d}{dt} \lambda_6 = (4\lambda_1^u + 2\lambda_2^u) \lambda_3^u + (4\lambda_1^d + 2\lambda_2^d) \lambda_3^d + 28\lambda_4 \lambda_6 + 8\lambda_4 \lambda_7 + 20\lambda_5 \lambda_6 + 4\lambda_5 \lambda_7 \\ + 4\lambda_6^2 + 2\lambda_7^2 + 16\lambda_6 \lambda_8 + 4\lambda_7 \lambda_8 + (-16\bar{g}_3^2 + 2\bar{Y}_{t_R}^2 + 2\bar{Y}_{b_R}^2 + 4\bar{Y}_{q_{3L}}^2) \lambda_6 \\ - 4\bar{Y}_{t_R}^2 \bar{Y}_{q_{3L}}^2 + \frac{11}{6} \bar{g}_3^4, \quad (41)$$

$$16\pi^2 \frac{d}{dt} \lambda_7 = 4\lambda_4 \lambda_7 + 8\lambda_5 \lambda_7 + 8\lambda_6 \lambda_7 + 6\lambda_7^2 + 4\lambda_7 \lambda_8 \\ + (-16\bar{g}_3^2 + 2\bar{Y}_{t_R}^2 + 2\bar{Y}_{b_R}^2 + 4\bar{Y}_{q_{3L}}^2) \lambda_7 + \frac{5}{2} \bar{g}_3^4, \quad (42)$$

$$16\pi^2 \frac{d}{dt} \lambda_8 = 2(\lambda_3^u)^2 + 2(\lambda_3^d)^2 + 6\lambda_6^2 + 4\lambda_6 \lambda_7 + 2\lambda_7^2 + 28\lambda_8^2 + (-16\bar{g}_3^2 + 8\bar{Y}_{q_{3L}}^2) \lambda_8 \\ - 4\bar{Y}_{q_{3L}}^4 + \frac{13}{6} \bar{g}_3^4. \quad (43)$$

Note that in all equations above we assumed real parameters. However, all formula can be easily generalized to the complex case by simply replacing a square by the absolute value squared.

By integrating these RGEs from M to the stop mass scale $m_{\tilde{t}}$, we obtain the $O(\alpha_3, Y_{t,b})$ contributions enhanced by $\log(M/m_{\tilde{t}})$.

D. Stop masses

In the effective theory, the stop mass matrix in the $(\tilde{t}_L, \tilde{t}_R)$ basis reads

$$\bar{\mathcal{M}}_t^2 = \begin{pmatrix} \bar{m}_{\tilde{Q}}^2 + v_u^2 \lambda_1^u + v_d^2 (\lambda_1^d + \lambda_2^d) & v_u \bar{A}_t^* - v_d \bar{\mu}_t^* \\ v_u \bar{A}_t - v_d \bar{\mu}_t & \bar{m}_{\tilde{t}}^2 + v_u^2 \lambda_3^u + v_d^2 \lambda_3^d \end{pmatrix}, \quad (44)$$

where $v_{u,d} = \langle H_{u,d}^0 \rangle$ are the vacuum expectation values of the Higgs scalars. By diagonalizing this matrix one obtains the stop masses and the stop mixing angle, both in the $\overline{\text{MS}}$ scheme. These masses are closely related to the left-handed sbottom mass

$$M_{\tilde{b}_L}^2 = \bar{m}_{\tilde{Q}}^2 + v_u^2 (\lambda_1^u + \lambda_2^u) + v_d^2 \lambda_1^d, \quad (45)$$

by $\text{SU}(2)$ gauge symmetry.

III. NUMERICAL ANALYSIS

From the previous analysis, we can see that, by integrating out the gluino and the squarks of the first two generations, parameters which were originally related via SUSY in the full MSSM, do not evolve anymore in the same way in the EFT. Let us illustrate this effect with two examples where striking differences between the EFT approach and the full MSSM emerge. Here we set the input parameters as $M = 5$ TeV, the stop mass scale $m_{\tilde{t}} = 500$ GeV, running top mass $m_t(m_{\tilde{t}}) = \bar{Y}_t(m_{\tilde{t}})v_u = 150$ GeV, $\alpha_3(m_{\tilde{t}}) = 0.1$, and $\tan \beta = v_u/v_d = 50$.

- The top Yukawa coupling Y_t

In the full MSSM, the Yukawa coupling Y_t of the superpotential enters top-top-Higgs, stop-stop-Higgs couplings as well as stop-squark-Higgsino couplings in the same way. However, in the EFT these couplings are independent quantities and they evolve differently below the scale M . This is depicted in Fig. 1, where the evolution of Y_t in the naive approach using MSSM RGE is compared to those of \bar{Y}_t , $\bar{Y}_{q_{3L}}$, \bar{Y}_{t_R} and $\bar{Y}_{\tilde{t}} \equiv \bar{\mu}_t/\bar{\mu}$ in the EFT. When the values of $\bar{Y}_{\tilde{t}}$ and Y_t are determined at the stop mass scale to give the SM running top mass, their values at the scale M are quite different.

- The quartic coupling of right-handed stops λ_8

In the full MSSM the quartic coupling of right-handed stops λ_8 is given by $\frac{1}{6}g_3^2$ by

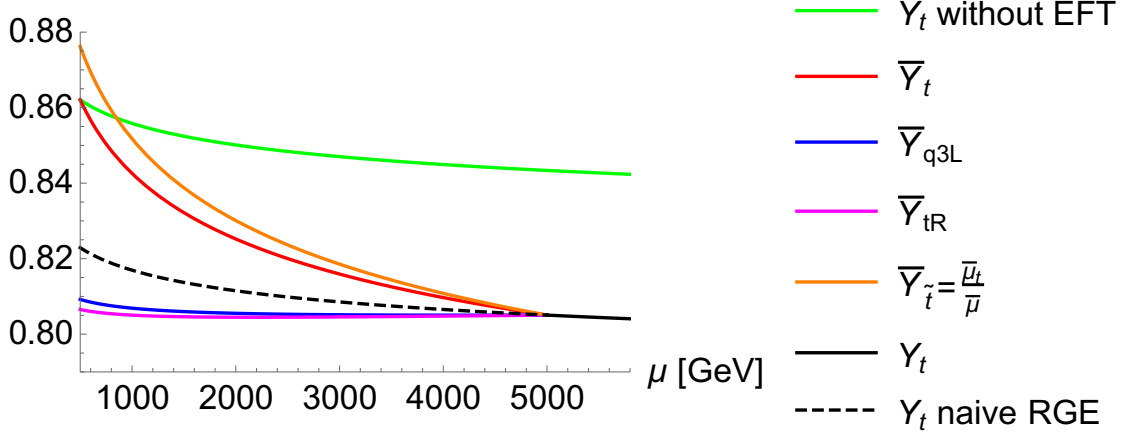


FIG. 1: Evolution of the Yukawa coupling Y_t in the naive approach without using an EFT (green) compared to the various Higgs/Higgsino-stop/top couplings in the EFT for $M = 5$ TeV and $\tan \beta = 50$ as a function of the renormalization scale μ . Note that the only numerically sizable impact of $\tan \beta = 50$ is the splitting between the \bar{Y}_{tR} and \bar{Y}_{q3L} . The initial condition of the Yukawa coupling is determined by the requirement that $v_u Y_t = m_t = 150$ GeV at the stop scale which we choose here to be 500 GeV. $\bar{Y}_{\tilde{t}} = \bar{\mu}_t / \bar{\mu}$ shows the evolution of the $\tilde{t} - \tilde{t} - H_d$ coupling relative to the Higgsino mass term $\bar{\mu}$ in the EFT. We also show the projected evolution of Y_t below the scale M (black-dashed) in the MSSM RGE for the boundary condition $Y_t(M) = \bar{Y}_t(M)$. Note that above the scale M SUSY is restored, so that there is only one Yukawa coupling Y_t (black).

SUSY relation and evidently also evolves in the same way as $\frac{1}{6}g_3^2$. However, in the EFT λ_8 and \bar{g}_3^2 follow different RGEs below the scale M , as seen in Fig. 2. The relative difference at the scale $m_{\tilde{t}}$ amounts to roughly 30%.

IV. CONCLUSIONS

In this article, we constructed an effective theory of the stop sector obtained from the full MSSM by integrating out the first and second generation of squarks and the gluino (which we assume to have a common mass of the order M). We computed the matching effects for the dimensionful quantities which are enhanced by powers of M at $O(\alpha_3, Y_{t,b}^2)$. In addition, we obtained the complete $O(\alpha_3, Y_{t,b}^2)$ RGEs of the couplings within the EFT. In the numerical analysis we highlighted that couplings which are related via SUSY identities within the full MSSM have different RGEs within the EFT, which can lead to sizable differences. We illustrated this effect for the top Yukawa couplings and the quartic coupling of right-handed stops, finding differences up to 30% between the EFT and the naive approach.

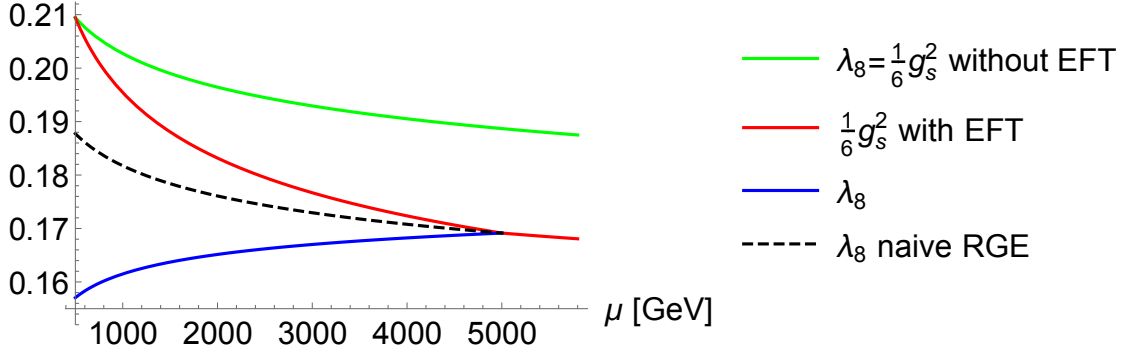


FIG. 2: Evolution of the quartic coupling to right-handed stops in the naive approach with the MSSM RGE (green) compared to the EFT approach, where $\alpha_3 = 0.1$ at the stop scale. Note that the SUSY relation $\lambda_8 = \frac{1}{6}g_3^2$ holds only at the scale M in the EFT. The dotted-black line shows the projected evolution of λ_8 for the boundary condition $\lambda_8(M) = \frac{1}{6}\bar{g}_s^2(M)$ with the naive RGE of the full MSSM. Note that above the scale M SUSY is restored, $\lambda_8 = 1/6g_s^2$ and evolves like g_s^2 in the full MSSM.

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Appendix Here we recall the RGEs of the parameters in the full MSSM, again taking into account $O(\alpha_3)$ and $O(Y_{t,b}^2)$ effects.

$$16\pi^2 \frac{d}{dt} g_3 = -3g_3^3, \quad (46)$$

$$16\pi^2 \frac{d}{dt} Y_t = Y_t \left[-\frac{16}{3}g_3^2 + 6Y_t^2 + Y_b^2 \right], \quad (47)$$

$$16\pi^2 \frac{d}{dt} Y_b = Y_b \left[-\frac{16}{3}g_3^2 + Y_t^2 + 6Y_b^2 \right], \quad (48)$$

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 6 \left(Y_t^2 (m_{H_u}^2 + m_{\tilde{Q}}^2 + m_{\tilde{t}}^2) + A_t^2 \right), \quad (49)$$

$$16\pi^2 \frac{d}{dt} m_{H_d}^2 = 6 (Y_b^2 (m_{H_d}^2 + m_{\tilde{Q}}^2 + m_{\tilde{b}_R}^2) + A_b^2), \quad (50)$$

$$16\pi^2 \frac{d}{dt} m_{H_d H_u}^2 = 3(Y_t^2 + Y_b^2) m_{H_d H_u}^2 + 6(Y_t A_t + Y_b A_b) \mu, \quad (51)$$

$$16\pi^2 \frac{d}{dt} m_{\tilde{Q}}^2 = -\frac{32}{3} g_3^2 m_{\tilde{g}}^2 + 2Y_t^2 (m_{\tilde{Q}}^2 + m_{H_u}^2 + m_{\tilde{t}}^2) \\ + 2Y_b^2 (m_{\tilde{Q}}^2 + m_{H_d}^2 + m_{\tilde{b}_R}^2) + 2(A_t^2 + A_b^2), \quad (52)$$

$$16\pi^2 \frac{d}{dt} m_{\tilde{t}}^2 = -\frac{32}{3} g_3^2 m_{\tilde{g}}^2 + 4Y_t^2 (m_{\tilde{Q}}^2 + m_{\tilde{t}}^2 + m_{H_u}^2) + 4A_t^2, \quad (53)$$

$$16\pi^2 \frac{d}{dt} m_{\tilde{b}_R}^2 = -\frac{32}{3} g_3^2 m_{\tilde{g}}^2 + 4Y_b^2 (m_{\tilde{Q}}^2 + m_{\tilde{b}}^2 + m_{H_d}^2) + 4A_b^2, \quad (54)$$

$$16\pi^2 \frac{d}{dt} \mu = 3(Y_t^2 + Y_b^2) \mu, \quad (55)$$

$$16\pi^2 \frac{d}{dt} A_t = A_t \left[-\frac{16}{3} g_3^2 + 18Y_t^2 + Y_b^2 \right] + 2Y_t Y_b A_b + \frac{32}{3} g_3^2 m_{\tilde{g}} Y_t, \quad (56)$$

$$16\pi^2 \frac{d}{dt} A_b = A_b \left[-\frac{16}{3} g_3^2 + Y_t^2 + 18Y_b^2 \right] + 2Y_t Y_b A_t + \frac{32}{3} g_3^2 m_{\tilde{g}} Y_b, \quad (57)$$

$$16\pi^2 \frac{d}{dt} m_{\tilde{g}} = -6g_3^2 m_{\tilde{g}}. \quad (58)$$

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